# From Dirichlet Forms to Wasserstein Geometry

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## Luigi Ambrosio (SNS Pisa)

### On some variational problems motivated by functions with bounded Hessian.

Motivated by problems related to deep learning, we study variational problems involving bounded Hessian functions, i.e. functions whose gradient is a function of bounded variation. In this context, particularly interesting are density results for piecewise affine functions and questions regarding extremal points. Work in progress with M.Unser, S.Aziznejad, C.Brena."

## Marc Arnaudon (University Bordeaux)

### Coupling of Brownian motions with set valued dual processes on Riemannian manifolds.

In this talk we will motivate and explain the evolution by renormalized stochastic mean curvature flow, of boundaries of relatively compact connected domains in a Riemannian manifolds. We will construct coupled Brownian motions inside the moving domains, satisfying a Markov intertwining relation. We will prove that the Brownian motions perform perfect simulation of uniform law, when the domain reaches the whole manifold. We will investigate the example of evolution of discs in spheres, and of symmetric domains in $\mathbb{R}^2$. Skeletons of moving domains will play a major role.

## Fabrice Baudoin (University Connecticut)

### Korevaar-Schoen-Sobolev spaces on metric measure spaces

We will review some of the recent developments in the theory of Sobolev spaces defined on general metric measure spaces.

## Mathias Beiglböck (University Vienna)

### The space of stochastic processes in continuous time.

Researchers from different areas have independently defined extensions of the usual weak topology between laws of stochastic processes. This includes Aldous' extended weak convergence, Hellwig's information topology and convergence in adapted distribution in the sense of Hoover-Keisler. We show that on the set of continuous processes with canonical filtration these topologies coincide and are metrized by a suitable adapted Wasserstein distance AW. Moreover we show that the resulting topology is the weakest topology that guarantees continuity of optimal stopping. While the set of processes with natural filtration is not complete, we establish that its completion consists precisely in the space processes with filtration FP. We also observe that (FP, AW) exhibits several desirable properties. Specifically, (FP, AW) is Polish, Martingales form a closed subset and approximation results like Donsker's theorem extend to AW.

## Lucian Beznea (University Bucharest)

### Continuous flows driving branching processes

We show that if the branching mechanism is specially constant then the superprocess is obtained by introducing the branching in the time evolution of the right continuous flow on measures, canonically induced by a right continuous flow as spatial motion. A corresponding result holds for non-local branching processes. The talk is based on a joint work with Catalin Ioan Vrabie from Bucharest.

## Mathias Braun (University Toronto)

### Timelike Ricci bounds for nonsmooth Lorentzian spaces by optimal transport.

We review recent breakthroughs in the synthetic description of timelike lower Ricci bounds for nonsmooth Lorentzian spaces, e.g. low regularity Lorentzian spacetimes. These bounds are encoded by convexity properties of certain entropy functionals along "chronological geodesics". We focus on the approach via Rényi's entropy, which parallels Sturm's celebrated CD condition for metric measure spaces, by the speaker. We obtain e.g. sharp timelike geometric inequalities, stability, and uniqueness of chronological optimal couplings and chronological geodesics. We also outline first nonsmooth examples of such so-called TCD spaces. Partly in collaboration with Matteo Calisti (Universität Wien).

## Krzysztof Burdzy (University Washington)

### From billiard balls to PDEs via white noise.

I was recently interested in the number of collisions of billiard balls in free space (with no walls). Tight configurations of balls seem to generate the highest number of collisions. "Pinned billiard balls" are tightly packed balls that do not move but they collide, in the sense that they have pseudo-velocities that evolve according to the law of totally elastic collision. Simulations suggest that the large-scale limit of the velocity process is "modulated white noise" whose parameters (mean and variance) satisfy "wave-like" PDEs.

## Fabio Cavalletti (SISSA Trieste)

### Isoperimetric inequality in noncompact spaces.

The sharp isoperimetric inequality for non-compact Riemannian manifolds with non-negative Ricci curvature and Euclidean volume growth has been obtained in increasing generality with different approaches in a number of contributions culminated by Balogh and Kristaly covering also m.m.s.'s verifying the non-negative Ricci curvature condition in the synthetic sense of Lott, Sturm and Villani. In contrast with the compact case of positive Ricci curvature, for a large class of spaces including weighted Riemannian manifolds, no complete characterisation of the equality cases was present in the literature. I will present how to settle this problem by proving, in the same generality of Balogh and Kristaly, that the equality in the isoperimetric inequality can be attained only by metric balls. Whenever this happens the space is forced, in a measure theoretic sense, to be a cone. Applications to the Euclidean anisotropic, weighted isoperimetric inequality will be discussed likewise. (Based on a joint paper with Davide Manini)

## Zhen-Qing Chen (University Washington)

### Stability of Elliptic Harnack Inequality

Harnack inequality, if it holds, is a useful tool in analysis and probability theory. In this talk, I will discuss scale invariant elliptic Harnack inequality for symmetric diffusion processes, or equivalently, for symmetric differential operators on metric measure spaces. We show that the elliptic Harnack inequality is stable under form-comparable perturbation for strongly local Dirichlet forms on complete locally compact separable metric spaces that satisfy metric doubling property.

Based on joint work with Martin Barlow and Mathav Murugan.

## Lorenzo Dello Schiavo (IST Austria)

### Conformally invariant random fields, quantum Liouville measures, and random Paneitz operators on Riemannian manifolds of even dimension

On large classes of closed even-dimensional Riemannian manifolds M, we construct and study the Copolyharmonic Gaussian Field, i.e. a conformally invariant log-correlated Gaussian field of distributions on M. This random field is defined as the unique centered Gaussian field with covariance kernel given as the resolvent kernel of Graham—Jenne—Mason—Sparling (GJMS) operators of maximal order. The corresponding Gaussian Multiplicative Chaos is a generalization to the 2m-dimensional case of the celebrated Liouville Quantum Gravity measure in dimension two. We study the associated Liouville Brownian motion and random GJMS operator, the higher-dimensional analogues of the 2d Liouville Brownian Motion and of the random Laplacian. Finally, we study the Polyakov–Liouville measure on the space of distributions on M induced by the copolyharmonic Gaussian field, providing explicit conditions for its finiteness and computing the conformal anomaly. (Based on arXiv:2105.13925, joint work with Ronan Herry, Eva Kopfer, and Karl-Theodor Sturm)

## Max Fathi (University Paris)

### Stability of the spectral gap on RCD spaces

A theorem of Lichnerowicz (1958) states that the spectral gap (or sharp Poincaré constant) of a smooth $n$-dimensional Riemannian manifold with curvature bounded from below by $n-1$ is bounded by $n$, which is the spectral gap of the unit $n$-sphere. This bound has since been extended to metric-measure spaces satisfying a curvature-dimension condition. In this talk, I will present a result on stability of the bound: if a space has almost minimal spectral gap, then the pushforward of the volume measure by a normalized eigenfunction is close to a Beta distribution with parameter $n/2$, with a sharp estimate on the L1 optimal transport distance. Joint work with Ivan Gentil and Jordan Serres

## Allessio Figalli (ETH Zürich)

### The singular set in the Stefan problem

The Stefan problem describes phase transitions such as ice melting to water, and it is among the most classical free boundary problems. It is well known that the free boundary consists of a smooth part (the regular part) and singular points. In this talk, I will describe a recent result with Ros-Oton and Serra, where we analyze the singular set in the Stefan problem and prove a series of fine results on its structure.

## Nicola Gigli (SISSA Trieste)

### Lipschitz continuity of harmonic maps from RCD to CAT(0) spaces

In 'classical' geometric analysis a celebrated result by Eells-Sampson grants Lipschitz continuity of harmonic maps from manifolds with Ricci curvature bounded from below to simply connected manifolds with non-negative sectional curvature. All these concepts, namely lower Ricci bounds, upper sectional bounds and harmonicity, make sense in the setting of metric-measure geometry and is therefore natural to ask whether the same sort of regularity holds in this more general setting. In this talk I will survey a series of recent papers that ultimately answer affirmatively to this question.

## Masha Gordina (University Connecticut)

### Limit laws and hypoellipticity

We will consider several classical problems for hypoelliptic diffusions and random walks: the large deviations principle (LDP), the small ball problem (SBP), Chung's law of iterated logarithm (LIL), and finding the Onsager-Machlup functional. As two very different examples we will look at hypoelliptic Brownian motion and the corresponding random walk on the Heisenberg group, and the Kolmogorov diffusion. We will explore the role of space-time scaling property, Gaussianity, and spectral properties via Dirichlet forms in these settings. The Onsager-Machlup functional is used to describe the dynamics of a continuous stochastic process, and it is closely related to the SBP and LIL, as well as the rate functional in the LDP. Unlike in the elliptic (Riemannian) case we do not rely on the tools from differential geometry such as comparison theorems or curvature bounds as these are not always available in the hypoelliptic (sub-Riemannian) setting. The talk is based on the joint work with Marco Carfagnini, Tai Melcher and Jing Wang.

## Alexander Grigor'yan (University Bielefeld)

### Finite propagation speed for Leibenson's equation on Riemannian manifolds

We consider on arbitrary Riemannian manifolds the Leibenson equation

$$\partial_t u = \Delta_p u^q.$$

This equation comes from hydrodynamics where it describes filtration of a turbulent compressible liquid in porous medium. We prove that if $p > 2$ and $1/(p-1) < q \leq 1$ then solutions of this equation have finite propagation speed. This work is joint with Philipp Sürig.

## Martin Grothaus (TU Kaiserslautern)

### Hypocoercivity for non-linear infinite-dimensional degenerate stochastic differential equations

Motivated by problems from Industrial Mathematics we further developed the concepts of hypocoercivity. The original concepts needed Poincaré inequalities and were applied to equations in linear finite dimensional spaces. Meanwhile we can treat equations in manifolds or even infinite dimensional spaces. The condition giving micro- and macroscopic coercivity we could relax from Poincaré to weak Poincaré inequalities. In this talk an overview and many examples are given.

## Martin Hairer (Imperial College London)

### The Brownian Castle and its crossover processes

Two natural Markov processes in 1+1 dimension arising as scaling limits of discrete systems with local interactions are the stochastic heat equation (or EW model) and the recently described KPZ fixed point. Here, we study a third such process which we call the Brownian Castle (BC). We also describe a class of processes arising from the BC -> EW crossover regime, showing in particular that the latter is non-universal, unlike the EW -> KPZ crossover.

## Shouhei Honda (University Tohoku)

### Topological stability theorem from nonsmooth to smooth spaces with Ricci curvature bounded below

Inspired by a recent work by Bing Wang and Xinrui Zhao, we prove that for a fixed closed $n$-dimensional Riemannian manifold $(M^n, g)$, if an $RCD(K, n)$ space $(X, d, m)$ is Gromov-Hausdorff close to $(M^n, d_g)$, then there exists a "canonical" homeomorphism $F$ from $X$ to $M^n$ such that $F$ is Lipschitz continuous and that $F^{-1}$ is Hölder continuous, where the Lipschitz constant of $F$, the Hölder exponent of $F^{-1}$ and the Hölder constant of $F^{-1}$ can be chosen arbitrary close to 1. Moreover if $(X, d)$ is smooth, then $F$ can be chosen as a diffeomorphism, which improves a result of Cheeger-Colding. This is a joint work with Yuanlin Peng (Tohoku University).

## Elton Hsu (Northwestern University)

### The Parisi Formula via Stochastic Analysis

The Parisi formula is a fundamental result in spin glass theory. In this talk I will present a new approach to (an enhanced Version of) Guerra's identity, from which the upper bound in the Parisi formula follows immediately. Among the techniques from stochastic analysis we will use include path space integration parts for the Wiener measure (Brownian motion), the Girsanov transform (exponential martingales), and the Feynman-Kac formula. The key observation is that the nonlinear partial differentiation equation figuring in Parisi's variation formula becomes linear after differentiating with respect to a parameter, thus allowing the full strength of stochastic analysis based on Ito's calculus into play. We hope that this approach will shed some lights on the much more difficult lower bound in the Parisi formula.

## Dirk Hundertmark (KIT Karlsruhe)

### Cwikel's bound reloaded.

There are several proofs by now for the famous Cwikel–Lieb–Rozenblum (CLR) bound, which is a semiclassical bound on the number of bound states for a Schrödinger operator, proven in the 1970s. Of the rather distinct proofs by Cwikel, Lieb, and Rozenblum, the one by Lieb gives the best constant, the one by Rozenblum does not seem to yield any reasonable estimate for the constants, and Cwikel's proof is said to give a constant which is at least about 2 orders of magnitude off the truth. This situation did not change much during the last 40+ years.

It turns out that this common belief, i.e, Cwikel's approach yields bad constants, is not set in stone: We give a substantial refinement of Cwikel's original approach which highlights a natural but overlooked connection of the CLR bound with bounds for maximal Fourier multipliers from harmonic analysis. Moreover, it gives an astonishingly good bound for the constant in the CLR inequality. Our proof is also quite flexible and leads to rather precise bounds for a large class of Schrödinger-type operators with generalized kinetic energies.

This is joint work with Peer Kunstmann (KIT), Tobias Ried (MPI Leipzig), and Semjon Vugalter (KIT)

## Jürgen Jost (MPI Leipzig)

### Nonpositive Curvature: Geometric and analytic aspects

In this talk, which represents joint work with Parvaneh Joharinad, I reconceptualize the notion of (generalized) sectional curvature in terms of intersection properties of balls. Thereby, it is linked to the notion of hyperconvexity and makes contact with constructions of topological data analysis.

## Nicolas Juillet (UHA Mulhouse)

### Exact interpolation of 1-marginals

I shall present a new type of martingales that exactly interpolates any given family of 1-dimensional marginals on $R^1$ (satisfying the suitable necessary assumption). The construction makes use of ideas from the (martingale) optimal transportation theory and relies on different stochastic orders. I shall discuss of related constructions and open questions (joint work with Brückerhoff and Huesmann).

## Moritz Kassmann (University Bielefeld)

### Function spaces and jump processes

The study of Markov jump processes and corresponding integro-differential operators has recently led to new developments in the theory of function spaces. Given a bounded domain in the Euclidean space, we explain how to formulate Dirichlet and Neumann problems for generators of jump processes that are allowed to jump into the complement of the domain. To this end we introduce nonlocal Sobolev-like function spaces and discuss their trace spaces. We define a nonlocal extension of the Dirichlet-to-Neumann map and provide a probabilistic interpretation of inhomogeneous nonlocal Neumann problems. The talk is based on research together with Bartek Dyda from Wroclaw University of Science and Technology (J. Funct. Anal. 2019), Guy Foghem from the Technische Universität Dresden (arXiv:2204.06793) and Soobin Cho from Seoul National University (ongoing work).

## Christian Ketterer (University Freiburg)

### Rigidity of the spectral gap for RCD spaces

Zhang and Yang proved a lower bound for the spectral gap of closed Riemannian manifolds with non-negative Ricci curvature. This bound is sharp, and by a result of Hang and Wang equality holds if and only if the space is isometric to the 1D circle with the same diameter.

In this talk I will survey rigidity results for the spectral gap of RCD spaces. The latter is the class of metric measure spaces with lower Ricci curvature bounds in the sense of Lott, Sturm and Villani that are infinitesimally Hilbertian. In particular, we will see a generalization of Hang and Wang's rigidity theorem. For the prove the 1D localization technique (using $L^1$-Wasserstein geometry) is combined with non-smooth Gamma calculus for the Dirichlet energy of the corresponding metric measure space. This is a joint work with Sajjad Lazian and Yu Kitabeppu.

## Motoko Kotani (University Tohoku)

### Discrete Surface Theory, and convergence of sequence of subdivided discrete surfaces

## Takashi Kumagai (University Kyoto)

### Periodic homogenization of non-symmetric jump-type processes with drifts

Homogenization problem is one of the classical problems in analysis and probability which is very actively studied recently. In this talk, we consider homogenization problem for non-symmetric Lévy-type processes with drifts in periodic media. Under a proper scaling, we show the scaled processes converge weakly to Lévy processes on $\mathbb{R}^d$. In particular, we completely characterize the limiting processes when the coefficient function of the drift part is bounded continuous, and the decay rate of the jumping measure is comparable to $r^{-1-\alpha}$ for $r > 1$ in the spherical coordinate with $\alpha \in (0, \infty)$. Different scaling limits appear depending on the values of $\alpha$.

This talk is based on joint work with Xin Chen, Zhen-Qing Chen and Jian Wang (Ann. Probab. 2021).

## Kazuhiro Kuwae (University Fukuoka)

### Liouville theorem for $V$-harmonic maps under non-negative $(m, V)$-Ricci curvature for non-positive $m$

Let $V$ be a $C^1$-vector field on an $n$-dimensional complete Riemannian manifold $(M, g)$. By using stochastic methods with geometric analysis, we talk on a Liouville theorem for $V$-harmonic maps satisfying various growth conditions from complete Riemannian manifolds with non-negative $(m, V)$-Ricci curvature for $m \in [-\infty, 0] \cup [n, +\infty]$ into Cartan-Hadamard manifolds, which extends S.Y. Cheng's Liouville theorem for sublinear growth harmonic maps from complete Riemannian manifolds with non-negative Ricci curvature into Cartan-Hadamard manifolds. This is a joint work with Songzi Li, Xiangdong Li and Yohei Sakurai.

## Xiang-Dong Li (PolyU Hong Kong)

### On the Entropy Power Inequality and Related Topics

In 1948, Claude Shannon established the mathematical foundation of information theory. In his classical paper, Shannon discovered the entropy power inequality. Since then, this inequality has received a lot of attentions in information theory, probability theory, convex geometry, differential geometry and related topics. In this talk, I will briefly recall the classical form of the entropy power inequality and its two proofs, then I will present its connection with probability theory, convex geometry and differential geometry. Finally, I will present our recent work on the entropy power inequality and the optimal transport problem, which leads to a new understanding to Lott-Villani and Sturm's synthetic geometry on the curvature-dimension condition on metric measure spaces.

## Jan Maas (IST Austria)

### Homogenisation of optimal transport on graphs

Many stochastic systems can be viewed as gradient flow in the space of probability measures, where the driving functional is a relative entropy and the relevant geometry is described by a dynamical optimal transport problem. In this talk we focus on these optimal transport problems and describe recent work on the limit passage from discrete to continuous.

Surprisingly, it turns out that discrete transport metrics may fail to converge to the expected limit, even when the associated gradient flows converge. We will illustrate this phenomenon in examples and present a recent homogenisation result.

This talk is based on joint works with Peter Gladbach, Eva Kopfer, and Lorenzo Portinale.

## Mattia Magnabosco (University Bonn)

### The Brunn-Minkowski inequality and its relation with the CD condition

The Lott-Sturm-Villani CD condition generalizes, to the setting of metric measure spaces, the notion of having Ricci curvature bounded from below and dimension bounded from above. One of the most important merits of the CD condition is that it is sufficient to deduce some geometric properties and functional inequalities that hold in the smooth world. A good example of this is the Brunn-Minkowski inequality, which, if properly generalized, is implied by the CD condition. In this talk I investigate whether the validity of the Brunn-Minkowski inequality is sufficient to prove the CD condition, showing in particular two results: the equivalence of the two requirements for weighted Riemannian manifolds and a slightly weaker equivalence result that holds in the general setting of metric measure spaces. This is a joint work with Lorenzo Portinale and Tommaso Rossi.

## Facundo Mémoli (Ohio State University)

### Gromov-like distances between spheres

Distances such as the Gromov-Hausdorff distance and its Optimal Transport variants are nowadays routinely invoked in applications related to data classification. Interestingly, the precise value of these distances on pairs of canonical shapes is known only in very limited cases. In this talk, I will describe lower bounds for the Gromov-Hausdorff distance between spheres (endowed with their geodesic distances) which we prove to be tight in some cases via the construction of optimal correspondences. These lower bounds arise from a certain version of the Borsuk-Ulam theorem for discontinuous functions.

## Emanuel Milman (Technion - Israel Institute of Technology)

### Isoperimetric Multi-Bubble Problems - Old and New

The classical isoperimetric inequality in Euclidean space $\mathbb{R}^n$ states that among all sets ("bubbles") of prescribed volume, the Euclidean ball minimizes surface area. One may similarly consider isoperimetric problems for more general metric-measure spaces, such as on the sphere $\mathbb{S}^n$ and on Gauss space $\mathbb{G}^n$. Furthermore, one may consider the "multi-bubble" partitioning problem, where one partitions the space into $q \geq 2$ (possibly disconnected) bubbles, so that their total common surface-area is minimal. The classical case, referred to as the single-bubble isoperimetric problem, corresponds to $q = 2$; the case $q = 3$ is called the double-bubble problem, and so on.

In 2000, Hutchings, Morgan, Ritoré and Ros resolved the Double-Bubble conjecture in Euclidean space $\mathbb{R}^3$ (and this was subsequently resolved in $\mathbb{R}^n$ as well) – the optimal partition into two bubbles of prescribed finite volumes (and an exterior unbounded third bubble) which minimizes the total surface-area is given by three spherical caps, meeting at $120°$-degree angles. A more general conjecture of J. Sullivan from the 1990's asserts that when $q \leq n + 2$, the optimal multi-bubble partition of $\mathbb{R}^n$ (as well as $\mathbb{S}^n$) is obtained by taking the Voronoi cells of $q$ equidistant points in $\mathbb{S}^n$ and applying appropriate stereographic projections to $\mathbb{R}^n$ (and backwards).

In 2018, together with Joe Neeman, we resolved the analogous multi-bubble conjecture on the optimal partition of Gauss space $\mathbb{G}^n$ into $q \leq n + 1$ bubbles – the unique optimal partition is given by the Voronoi cells of (appropriately translated) $q$ equidistant points. In this talk, we will describe our approach in that work, as well as recent progress on the multi-bubble problem on $\mathbb{R}^n$ and $\mathbb{S}^n$. In particular, we show that minimizing partitions are always spherical when $q \leq n+1$, and we resolve the latter conjectures when in addition $q \leq 6$ (e.g. the triple-bubble conjecture in $\mathbb{R}^3$ and $\mathbb{S}^3$, and the quadruple-bubble conjecture in $\mathbb{R}^4$ and $\mathbb{S}^4$).

Based on joint work with Joe Neeman.

## Andrea Mondino (University Oxford)

### Unification of Riemannian, Finslerian and Sub-Riemannian synthetic Ricci lower bounds

In his 2017 Bourbaki seminar, Villani proposed as open problem to obtain a "Great unification" of synthetic Riemannian, Finslerian and Sub-Riemannian synthetic Ricci lower bounds. The goal of the talk is to report on joint work with Barilari (Padova) and Rizzi (SISSA) on the topic.

## Shin-ichi Ohta (University Osaka)

### Discrete-time gradient flows in Gromov hyperbolic spaces

The theory of gradient flows for convex functions on "Riemannian" spaces (such as CAT(0)-spaces) has been making impressive progress. Nonetheless, much less is known for "Finsler" spaces, especially the lack of contraction property is a central problem. As a class including some non-Riemannian Finsler manifolds, we employ Gromov hyperbolic spaces and study the contraction property of discrete-time gradient flows for convex functions. This talk is based on a preprint available at arXiv:2205.03156.

## Felix Otto (MPI Leipzig)

### Regularity structures without Feynman diagrams

Singular stochastic PDE are those stochastic PDE in which the noise is so rough that the nonlinearity requires a renormalization. Hairer's regularity structures provide a framework for the solution theory. His notion of a model can be understood as providing a (formal) parameterization of the entire solution manifold of the renormalized equation. In this talk, I will focus on the stochastic estimates of the model.

I shall present a more analytic than combinatorial approach: Instead of using trees to index the model, we consider all partial derivatives w. r. t. the function defining the nonlinearity (and thus work with multi-indices as index set). Instead of a Gaussian calculus guided by Feynman diagrams arising from pairing of trees, we consider first-order partial derivatives w. r. t. the noise, i. e. Malliavin derivatives.

We employ tools from quantitative stochastic homogenization like spectral gap estimates, which naturally complement the standard choice of renormalization, and annealed estimates, which as opposed to their quenched counterparts preserve scaling. The gain in regularity when taking a Malliavin derivative, and thus replacing one instance of the noise by an element of the Cameron-Martin space, is conveniently captured in terms of a modelled distribution.

This is joint work with P. Linares, M. Tempelmayr, and P. Tsatsoulis, based on work with J. Sauer, S. Smith, and H. Weber.

## Miklós Pálfia (Corvinus University Budapest)

### Sturm's law of large numbers for the $L^1$-Karcher mean of positive operators

Firstly we briefly review some available versions of the strong law of large numbers and nonlinear extensions provided by Sturm in CAT(0) metric spaces. Sturm's 2001 $L^2$-result was directly applied to the case of the geometric (also called Karcher) mean of positive matrices, thus it suggests a natural formulation of the law for positive operators. However there are serious obstacles to overcome to prove the law in the infinite dimensional case. We propose to use a recently established gradient flow theory by Lim-P for the Karcher mean of positive operators and a stochastic proximal point approximation to prove the $L^1$-strong law of large numbers for the Karcher mean in the operator case.

## Gabriel Peyré (ENS Paris)

### Unbalanced Optimal Transport across Metric Measured Spaces

Optimal transport (OT) has recently gained a lot of interest in machine learning. It is a natural tool to compare in a geometrically faithful way probability distributions. It finds applications in both supervised learning (using geometric loss functions) and unsupervised learning (to perform generative model fitting). OT is however plagued by several issues, and in particular: (i) the curse of dimensionality, since it might require a number of samples which grows exponentially with the dimension, (ii) sensitivity to outliers, since it prevents mass creation and destruction during the transport (iii) impossibility to transport between two disjoint spaces. In this talk, I will review several recent proposals to address these issues, and showcase how they work hand-in-hand to provide a comprehensive machine learning pipeline. The three key ingredients are: (i) entropic regularization which defines computationally efficient loss functions in high dimensions (ii) unbalanced OT, which relaxes the mass conservation to make OT robust to missing data and outliers, (iii) the Gromov-Wasserstein formulation, introduced by Sturm and Memoli, which is a non-convex quadratic optimization problem defining transport between disjoint spaces. More information and references can be found on the website of our book "Computational Optimal Transport" https://optimaltransport.github.io/

## Tapio Rajala (University Jyväskylä)

### Tensorization of Sobolev spaces

I will present new results on the tensorization problem of first order Sobolev spaces defined on metric measure spaces. For a general Sobolev exponent the tensorization of weak differentiable structures leads to a tensorization result where one of the factors is assumed to be a PI-space. When the Sobolev exponent is 2 we can make use of the tensorization property of Dirichlet forms and obtain the tensorization for spaces where the Sobolev norm is equivalent to a norm given by a Dirichlet form. Such spaces include infinitesimally Hilbertian spaces and spaces with finite Hausdorff dimension. This is joint work with Sylvester Eriksson-Bique and Elefterios Soutanis.

## Michael Röckner (University Bielefeld)

### On a longstanding open problem in the theory of Markov processes

We define a class of not necessarily linear $C_0$-semigroups $(P_t)_{t\geq 0}$ on $C_b(E)$ (more generally, on $C_\kappa(E) := \frac{1}{\kappa}C_b(E)$, for some growth bounding continuous function $\kappa$) equipped with the mixed topology $\tau_1^{\mathscr{M}}$ for a large class of topological state spaces $E$. In the linear case we prove that such $(P_t)_{t\geq 0}$ can be characterized as integral operators given by measure kernels satisfying certain properties. We prove that the strong and weak infinitesimal generators of such $C_0$-semigroups coincide. As a main result we prove that transition semigroups of Markov processes are $C_0$-semigroups on $(C_b(E), \tau_1^{\mathscr{M}})$, if they leave $C_b(E)$ invariant and they are jointly weakly continuous in space and time. In particular, they are infinitesimally generated by their generator $(L, D(L))$ and thus reconstructable through an Euler formula from their strong derivative at zero in $(C_b(E), \tau_1^{\mathscr{M}})$. This solves a long standing open problem on Markov processes. Our results apply to a large number of Markov processes given as the laws of solutions to SDEs and SPDEs, including the stochastic 2D Navier-Stokes equations and the stochastic fast and slow diffusion porous media equations. Furthermore, we introduce the notion of a Markov core operator $(L_0, D(L_0))$ for the above generators $(L, D(L))$ and prove that uniqueness of the Fokker-Planck-Kolmogorov equations corresponding to $(L_0, D(L_0))$ for all Dirac initial conditions implies that $(L_0, D(L_0))$ is a Markov core operator for $(L, D(L))$. As a consequence we can identify the Kolmogorov operator of a large number of SDEs on finite and infinite dimensional state spaces as Markov core operators for the infinitesimal generators of the $C_0$-semigroups on $(C_\kappa(E), \tau_1^{\mathscr{M}})$ given by their transition semigroups. If each $P_t$ is merely convex, we prove that $(P_t)_{t\geq 0}$ gives rise to viscosity solutions to the Cauchy problem of its associated (non linear) infinitesimal generators. Furthermore, we prove that each $P_t$ has a stochastic representation as a convex expectation in terms of a nonlinear Markov process.

Joint work with: Ben Goldys (University of Sydney), Max Nendel (Bielefeld University)

## Giuseppe Savaré (University Milan)

### Density of subalgebras of Lipschitz functions in metric Sobolev spaces and applications to measured Wasserstein spaces

We will present some results on Sobolev spaces in metric-measure spaces, in particular regarding approximation by subalgebras of Lipschitz functions.

As an application, we will show that the Dirichlet form obtained by relaxation of the L2-Sobolev energy associated with a finite Borel measure on the Euclidean L2-Wasserstein space and the corresponding gradient of cylindrical functions coincides with the Cheeger energy arising from the integration of the squared asymptotic Lipschitz constant of Lipschitz functions.

As a byproduct, we derive the infinitesimal Hilbertianity of the L2-Wasserstein space in Riemannian manifolds or Hilbert spaces.

## Rene Schilling (TU Dresden)

### The Liouville Property For Generators of Levy Processes

We give a purely analytic proof, leading to necessary and sufficient conditions, for the Liouville property for generators of Levy processes. Using probabilistic arguments we show a version of the strong Liouville property. Finally, we explore the connection to coupling of Levy processes.

## Takashi Shioya (University Tohoku)

### Principal bundle structure of the space of metric measure spaces

We study the topological structure of the space $\mathcal{X}$ of isomorphism classes of metric measure spaces equipped with the box and concentration topologies. We consider the scale-change action of the multiplicative group $R_+$ of positive real numbers on $\mathcal{X}$, which has a one-point metric measure space, say $*$, as a unique fixed point. One of our main theroem states that the $R_+$-action on $\mathcal{X} \setminus \{*\}$ admits the structure of nontrivial and locally trivial principal $R_+$-bundle over the quotient space. This is a joint work with Daisuke Kazukawa (Kyushu Univ.) and Hiroki Nakajima (Tohoku Univ.).

## Wilhelm Stannat (TU Berlin)

### Mean-field approach to Bayesian estimation of Markovian signals

Estimating Markovian signals X from noisy observations is an important problem in the natural and engineering sciences. Within the Bayesian approach the underlying mathematical problem essentially consists in the (stochastic) analysis of the conditional law of X with a view towards its efficient numerical approximation.

In this talk I will discuss mean-field type descriptions of the conditional law of X, when X is the solution of a stochastic differential equation, and present recent results on corresponding ensemble-based numerical approximations in the case with correlated observation noise.

The talk is based on joint work with S. Ertel, S. Pathiraja and S. Reich.

## Peter Stollmann (TU Chemnitz)

### Essential spectrum and Feller type properties

We report on recent joint work with A. Ben Amor and B. Güneysu that gives a decomposition principle for Dirichlet forms under conditions that are partly weaker than the earlier result by Lenz and Stollmann. In particular, our results apply to Cheeger forms on RCD$^*$ spaces.

## Wolfgang Stummer (FAU Erlangen)

### Optimal Transport with Some Directed Distances.

We present a toolkit of directed distances between quantile functions. By employing this, we solve some new optimal transport (OT) problems which e.g. considerably flexibilize some prominent OTs expressed through Wasserstein distances.

## Kohei Suzuki (University Bielefeld)

### On the Geometry of Configuration Spaces and Particle Systems

The configuration space U(X) over a base space X is the space of all locally finite point measures on X. The space U(X) being equipped with the vague topology, the $L^2$-transportation distance and a point process, it is a Polish extended metric measure space. In this talk, we show that U(X), equipped with the Poisson point process, satisfies synthetic lower Ricci curvature bounds if and only if so does X. As a byproduct, we obtain the Sobolev-to-Lipschitz property on U(X), which confirms the conjecture by Röckner-Schied (J. Func. Anal. '99). We discuss several applications to the corresponding infinite-particle systems such as the integral Varadhan short-time asymptotic of the heat flow on U(X) and a new characterisation of ergodicity of particle systems in terms of the $L^2$-transportation distance. If time allows, we also explain the case beyond the Poisson point process. This talk is based on the joint work with Lorenzo Dello Schiavo (Institute of Science and Technology Austria).

## Asuka Takatsu (Tokyo Metropolitan University)

### Concavity properties preserved by the Dirichlet heat flow

In 1976, Brascamp and Lieb proved that the Dirichlet heat flow preserves log-concavity. In this talk, we introduce a variation of concavity to complete the study of preservation properties by the Dirichlet heat flow. This talk is based on the joint work with Kazuhiro Ishige (the University of Tokyo) and Paolo Salani (Universita di Firenze).

## David Tewodrose (University Nantes)

### Kato limit spaces

In this talk, I will present a couple of joint works with Gilles Carron (Nantes Université) and Ilaria Mondello (Université Paris Est Créteil) where we study geometric and analytic properties of Kato limit spaces, which are measured Gromov-Hausdorff limits of closed Riemannian manifolds with negative part of the greatest pointwise lower bound of the Ricci curvature in a uniform Kato class. This assumption allows for the Ricci curvature to degenerate to - infinity, but in a way that is controlled by the heat kernel.

## Anton Thalmaier (University Luxembourg)

### Higher order derivative formulae for heat semigroups and Calderón-Zygmund inequalities on Riemannian manifolds.

We discuss new formulae for the Hessian of heat semigroups generated by the Laplace-Beltrami operator on a Riemannian manifold, possibly with boundary. We give various geometric applications of these formulae, including explicit Hessian estimates for Dirichlet and Neumann eigenfunctions, as well as applications related to Riesz transforms, Calderón-Zygmund type inequalities and log-Sobolev inequalities. Our approach relies on probabilistic methods from Stochastic Analysis.

Based on joint work with Jun Cao, Li-Juan Cheng, Feng-Yu Wang.

## Gerald Trutnau (Seoul National University)

### Existence and uniqueness of (infinitesimally) invariant measures for second order partial differential operators on Euclidean space

We consider a locally uniformly strictly elliptic second order partial differential operator in $\mathbb{R}^d$, $d \geq 2$, with low regularity assumptions on its coefficients, as well as an associated Hunt process and semigroup. The Hunt process is known to solve a corresponding stochastic differential equation that is pathwise unique. In this situation, we study the relation of invariance, infinitesimal invariance, recurrence, transience, conservativeness and $L^r$-uniqueness. Our main result is that recurrence implies uniqueness of infinitesimally invariant measures, as well as existence and uniqueness of invariant measures. We can hence make in particular use of various explicit analytic criteria for recurrence that have been previously developed in the context of (generalized) Dirichlet forms and present diverse examples and counterexamples for uniqueness of infinitesimally invariant, as well as invariant measures and an example where $L^1$-uniqueness fails although pathwise uniqueness holds. Furthermore, we illustrate how our results can be applied to related work and vice versa. This is joint work with Haesung Lee.

## Feng-Yu Wang (Beijing Normal University)

### Wasserstein Limit for Empirical Measures of Diffusion Processes

The limit in Wasserstein distance is presented by using eigenvalues and eigenfunctions for the empirical measures of diffusion processes on compact Riemannian manifolds or a bounded domain with reflecting or killing boundary. The convergence rate is estimated also for SDEs and semilinear SPDEs.